

**RELIABILITY CALIBRATION OF A WAVES- ICEBERGS INTERACTION LOAD COMBINATION FACTOR**

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**ABSTRACT**

The Canadian Standards Association [1] has developed and published a code for the design and construction of fixed offshore structures. One of the limit states relates to the combined effects of waves and iceberg collision loading. The Code uses a load combination factor to determine the design load effect. The present paper describes a recent study on the appropriateness of the recommended value of the combination factor. The study involves a numerical analysis in which loads have been calculated, at different probability levels, for a range of iceberg and wave parameters, considering waves alone, an iceberg alone, and an iceberg and waves in combination,. The paper thereby makes recommendations for the load combination factor as a function of iceberg and sea state parameters.

**INTRODUCTION**

The selection of suitable environmental loads and load events is of critical importance in the design of offshore structures in extreme environments. Such loads may include effects of wind,

waves, earthquakes, ice and iceberg collisions. The CSA Offshore Structures Code CAN/CSA-S471-92(S471) [1] , that is currently in use, indicates the use of probabilistic methods on which the selection of load events and design loads should be based. One of the important limit states is the combined effect of wave and iceberg collision loading. This is treated by the use of a load combination factor specified in the Code. The present paper describes a recent study that was undertaken to determine the appropriateness of Code recommendations.

Wave-structure interactions (in the absence of icebergs) has been studied extensively (e.g., [2]). In the case of large offshore structures, linear diffraction wave theory is generally used to calculate wave loads. This is based on the assumption of potential flow theory, a horizontal seabed and small amplitude waves. Structures of arbitrary shape are usually treated by a boundary element method in which the submerged surface of the structure is specified in discretized form. Iceberg-structure interactions (in the absence of waves) have also been studied extensively, and an overview of this topic has been given by Cammaert and Muggeridge [3]. In general, the maximum iceberg load on a structure is obtained by an energy balance in which the initial kinetic energy of the iceberg is equated to the energy dissipated through ice crushing up to the time the iceberg is brought to rest. Topics of particular interest in this regard include the consideration of energy dissipation through structural damage

or ductility, and an assessment of the importance of size (area) effects on the ice-crushing pressure.

Wave effects on iceberg motions and the case of waves and an iceberg acting simultaneously on an offshore structure have also been widely studied, although perhaps not to the same extent. Particular aspects which have been considered include wave-induced iceberg motions (e.g. [4] and [5]), iceberg interactions with semi-submersible rigs (e.g. [6] and [7]), and iceberg motions near a large structure (e.g. [8] and [9]). Isaacson [8] considered the effect of waves on an iceberg up to the instant of impact with a large structure, and described a numerical model for evaluating iceberg drift motions in order to provide an assessment of wave effects on the iceberg velocity and effective mass at the time of impact.

The present paper describes a numerical analysis in which loads due to waves alone, an iceberg alone, and an iceberg and waves in combination, have been calculated for a range of iceberg and wave parameters. These results have been used to develop expressions for wave and iceberg loads which are then used in a probabilistic study of the load event. The probability assessment is based on the first-order reliability method (FORM). As a case study, the paper analyses conditions similar to those of the Hibernia platform located in the Grand Banks off the Newfoundland coast. This platform is a gravity-based reinforced concrete structure, protected from iceberg impact by a concrete cylindrical wall. In this study, the structure is assumed to be a vertical circular cylinder of radius  $a = 58$  m located in a water depth  $d = 80$  m.

## PROBABILISTIC FRAMEWORK

Probabilistic analyses have previously been used in offshore engineering problems (e.g. [10], [11] and [12]) including the calibration of the Canadian Code for offshore structures [13]. In the present study, however, the probabilistic framework includes the formulation of detailed mechanical and hydrodynamic models for the interaction of waves and icebergs during a collision. Thus, in what follows, a detailed description is given on the approaches used to assess the loads due to waves alone, an iceberg alone, and waves and an iceberg acting in combination.

The estimation of conditional probabilities associated with a load event is conducted using the program RELAN (RELIability ANalysis), developed at the Civil Engineering Department of the University of British Columbia [14]. This program implements standard FORM algorithms (First Order Reliability Methods) to calculate the probability that a "performance function"  $G$  of the vector of a set of random variables  $x$  is negative. In the present context, in order to compute the exceedence probability of the load level  $F$ , the function  $G(x)$  is written as follows:

$$G(x) = F - F_M(x) R_n \quad (1)$$

in which  $F_M(x)$  is the maximum force developed on the structure due to waves, or iceberg impact, or waves and iceberg impact in combination, as appropriate,  $x$  denotes a set of specified random variables characterizing the structure, the iceberg and the wave conditions, and  $R_n$  is a random variable associated with model inaccuracy in the calculation of  $F_M$  [15].

The probability of the event  $G < 0$  corresponds to the probability that the maximum load  $F_M$  exceeds the load level  $F$ . In the case of an iceberg impact in the absence of waves, the force exceedence probability is first obtained conditional on the occurrence of an impact. In such a case, the programs allow for the estimation of the corresponding annual risk, denoted  $p_a$ , using the hypothesis that the events (i.e. iceberg impacts) follow a Poisson pulse process with a given mean rate of annual occurrence (events per year), denoted  $\mu$ . Thus, if the conditional exceedence probability of the event is  $p_e$ , the annual risk is given as:

$$p_a = 1 - \exp(-\mu p_e) \quad (2)$$

The mean rate of collision events,  $\mu$ , has recently received some attention (e.g. [16]). For the purpose of this work,  $\mu$  has been assumed to take values ranging from 0.04 to 1.00.

In the case of waves alone, the force exceedence annual risk is directly obtained using annual maxima distributions for the sea state. In the case of an iceberg impact in the presence of waves, the force exceedence probability is first obtained conditional on the occurrence of an impact, so that Eq. (2) is also needed to transform this conditional probability into the corresponding annual risk. In this case, the probability distributions for the wave parameters correspond to the sea state that is present at the time of impact, and are based on measured records of wave periods and heights at a specified recording interval  $\tau$ , taken here as six hours.

In each of the three cases above, the maximum force  $F_M$  developed during the event requires an appropriate mechanics model for its calculation. The following sections describe, briefly, such calculations for waves alone, an iceberg alone, and waves and an iceberg in combination.

## FORCES DUE TO WAVES ALONE

An analysis is first carried out for wave loads in the absence of icebergs. A simple representation of the maximum annual distribution of wave conditions is initially required, and a single parameter, the peak period  $T$ , is used here. The maximum annual wave period is assumed to obey an Extreme Type I annual probability distribution (e.g. [2]), such that  $T = 18$  sec corresponds to a cumulative probability  $p(T) = 0.99$  (return period  $T_R = 100$  years); and  $T = 15$  sec to  $p = 0.05$  ( $T_R = 1.05$  years). Thus, in general,

$$p(T) = \exp[-\exp(-1.8991(T - 15.5777))] \quad (3)$$

where  $T$  is in seconds. The significant wave height  $H_s$  (in m) can be estimated from a formula which has been found to be suitable for conditions in Canadian Atlantic waters [17]:

$$H_s = 0.0509 T^2 \quad (4)$$

The loads for regular waves may be obtained from an analysis based on linear diffraction theory. Results are obtained using a

computer program WELSAS, which is based on three-dimensional linear diffraction theory using a boundary element method [2]. In the case of a single fixed structure, the flow is described by a velocity potential  $\Phi$  comprised of a component  $\Phi_W$  associated with the incident waves, which is known, and a component  $\Phi_S$  associated with the scattered waves, which is to be determined:

$$\Phi = \Phi_W + \Phi_S \quad (5)$$

The latter satisfies the Laplace equation within the fluid region, linearized kinematic and dynamic boundary conditions at the still water surface, a radiation condition in the far field, and kinematic conditions at the seabed and the submerged structure surface. Such a potential may be represented as due to a distribution of 'point wave sources' over the submerged surface of the structure:

$$\Phi_S(x) = \int_S f(\xi) G(x, \xi) dS \quad (6)$$

where  $f(\xi)$  represents a source strength distribution function,  $x$  is a general point  $(x, y, z)$  within the fluid region, and the integral is taken over the points  $\xi$  on the submerged surface of the body,  $S$ .  $G(x, \xi)$  is a known Green's function and corresponds to the potential at the point  $x$  due to a source of unit strength at  $\xi$ . The application of the boundary condition on the submerged structure surface gives rise to an integral equation for the unknown source strength distribution function  $f(\xi)$  on this surface. The submerged structure surface is discretized into a finite number of facets, such that the source distribution is approximated by point sources at the facet centers, and the integral equation is thereby approximated by a matrix equation for the source strengths. This is solved to provide the source strengths, and the potentials at the facet centers are then obtained by a discretized version of Eq. 6. Finally, the pressure distribution around the structure may be expressed in terms of the corresponding velocity potential, and the force and moment components on the structure are then obtained by suitable integrations of the pressure distribution. These are used to provide the maximum horizontal force  $F_H$ , the maximum vertical force  $F_V$ , and the maximum overturning moment  $M_O$ . As input to the program, the range and number of wave frequencies and wave directions is required, and the submerged configuration of the structure is specified in a discretized form. For each wave period and direction, the linear diffraction problem is solved to provide the various force components indicated above. For regular waves of height  $H$  and period  $T$ , the force on the structure varies sinusoidally with period  $T$ , and thus is expressed as  $F \cos(\omega t)$ , where  $\omega = 2\pi/T$  and  $F$  is the force amplitude.  $F$  is proportional to the wave height, so that results need only be obtained for waves of unit height. Since the wave forces are ultimately required for application in a probabilistic model, a simple expression has been fitted to numerical results obtained for wave periods ranging from  $T = 10$  to 20 sec. The results correspond to a water depth of 80 m, a water density of  $1,025 \text{ kg/m}^3$ , and a structure of radius 58 m. Thus, for the case of regular waves alone, the maximum force  $F_M = F$  on the structure is obtained in the form:

$$F = H[-136.807 + 22.546T - 0.593T^2] \quad (7)$$

where  $F$  is in MN.

The loads due to random waves may be obtained as an extension of the case of regular waves. However, the height  $H$  is now random within a storm and obeys a Rayleigh distribution,

$$p(H) = R_n = 1 - \exp\left[-2\left(\frac{H}{H_s}\right)^2\right] \quad \text{for } H \geq 0 \quad (8)$$

where  $R_n$  is a random variable uniformly distributed between 0 and 1. For a pair of random wave height and period values,  $H$  and  $T$ , the corresponding force may be obtained from Eq. (7).

The resulting force  $F$  would correspond to the random individual values within the maximum annual storm, while what is required is the maximum  $F$  within that storm. Thus, the distribution for the maximum  $F$  depends on the storm duration  $\tau$ , with the number of waves within that storm taken as  $N = 3600 \tau / T$ , in which  $\tau$  is in hours and  $T$  is in seconds. Results are given here for storm durations of 4, 8 and 16 hours.

## FORCE DUE TO ICEBERG COLLISION ALONE

The force developed during an iceberg collision will vary during the process of ice crushing against the structure, since the area of contact is continuously changing and the crushing pressure exhibits a notable size effect (the greater the contact area, the lower the required pressure). The force is also influenced by the damage deformation of the structure, which may have been designed to allow for local damage when the force exceeds a certain level. Here, local damage is associated with the deformation or collapse of the structure in the neighborhood of the contact point with the iceberg. It is quite difficult to derive relationships between applied pressure and structural damage during a situation of progressive collapse.

The necessary background of ice mechanics and risks to offshore structures have been amply discussed elsewhere by Sanderson [18]. The present study applies such basic concepts to the example under consideration here, and extends the formulation to take account of iceberg penetration due to local structural damage. The size effect in ice crushing pressure is also considered.

## ICEBERG SHAPE AND SIZE

It is very difficult to represent accurately the three-dimensional shape of an iceberg by a simple mathematical equation. The approach adopted here follows that of Det Norske Veritas [11] in which the iceberg is assumed to be circular in plan and ellipsoidal in elevation, Fig. 1, with horizontal (major) and vertical (minor) semi-axes  $R$  and  $B$  respectively. From statistical data for the Grand Banks, all the iceberg dimensions are expressed in terms of a single random variable  $L$  (in m) which is represented by a Gamma distribution, with a mean value  $\mu_L = 121.60$  m and a standard deviation  $\sigma_L = 56.70$  m. Other characteristic dimensions of the iceberg may be expressed in terms of  $L$ . In

particular, the horizontal semi-axis R, and the iceberg diameter at the waterline D are given respectively as:

$$R = 0.428 L + 1.053 L^{0.63} \quad (9)$$

$$D = 0.679 L + 1.671 L^{0.63} \quad (10)$$

It can be shown that the vertical semi-axis B, and the iceberg height above the water, b, are related to the draft h according to:

$$B = \frac{h}{1.608} \quad (11)$$

$$b = 0.244 h \quad (12)$$

The iceberg draft h is in turn related to L, except that icebergs capable of colliding must have a draft smaller than the water depth of 80 m. Thus:

$$h = \text{Min} \begin{cases} 3.781L^{0.63} \\ d \end{cases} \quad (13)$$

Using the specified Gamma distribution for L, Eq. (13) is used to obtain a corresponding distribution of the draft h. Taking account of the truncation at a maximum of 80 m, the data for h have been represented with a Beta distribution with a minimum of 0 m and a maximum of 80 m, resulting in a mean draft  $\mu_h = 61.35$  m with a standard deviation  $\sigma_h = 12.38$  m.

#### ICE CRUSHING PRESSURE

For different penetrations x into the ice, as shown in Fig. 1, it is possible to compute the area of contact as the intersection of the ellipsoid with the cylindrical structure of radius a. From a knowledge of the relationship between ice-crushing pressure and area, the force F(x) may then be obtained by integration, assuming that the pressure is uniformly distributed over the area. The impact is assumed to be head-on, and the contact area is computed accordingly. The possibility of eccentric impact is taken into account through the use of a modification factor which is subsequently described.

The pressure p required to crush the ice depends on the area of contact A. It is assumed that the crushing pressure p has a lognormal distribution, with mean  $m_p$  and coefficient of variation  $V_p$ :

$$p = \frac{m_p}{\sqrt{1 + V_p^2}} \exp \left[ R_{n4} \sqrt{\ln(1 + V_p^2)} \right] \quad (14)$$

where  $R_{n4}$  has a Standard Normal distribution.

In general, the size effect for the mean pressure  $m_p$  can be written in the form:

$$m_p = \text{Max.} \begin{cases} C_1 A^{C_2} \\ p_0 \end{cases} \quad (15)$$

Although data on iceberg ice are scarce, the scatter in available data for ice in Arctic conditions [19] is reasonably well represented with the following values:

$$\begin{aligned} V_p &= 0.50 \\ C_1 &= 9.0 \text{ MPa} \\ C_2 &= -0.5 \\ p_0 &= 2.0 \text{ MPa} \end{aligned} \quad (16)$$

with A in  $m^2$ . The value  $p_0$  is a lower bound for  $m_p$ . Due to uncertainty in ice crushing pressure for large areas A, it may be more appropriate to represent  $p_0$  by a suitable probability distribution. Instead, in the present study  $p_0$  is taken as a constant. It should be noted that the lower bound  $p_0 = 2$  MPa is reached at a contact area of about  $20 m^2$ , which is probably very quickly exceeded during a collision. The value of  $p_0$  is not well defined from available data, and since it is expected to have marked influence on the loads developed during the collision, three specific values of  $p_0$  are studied: 2, 4 and 6 MPa.

#### FORCE-PENETRATION RELATIONSHIP

For a given penetration x due to ice crushing, Fig. 1, the force F(x) acting on the structure can be calculated from an integration of the crushing pressure p over the area of contact A(x):

$$A(x) = 2a \int_0^{\alpha} s(x, \psi) d\psi \quad (17)$$

$$F(x) = 2a p(x) \int_0^{\alpha} s(x, \psi) \cos\psi d\psi \quad (18)$$

where the angles  $\psi$  and  $\alpha$  are shown in Fig. 1, and s is the height of the contact area corresponding to the angle  $\psi$ . In general, it is found that the iceberg will be stopped after a few meters of penetration. The relationship F(x) may thus be linearized and, in the present case, this linearization is achieved by replacing the curve F(x) with a straight line fit to results for penetrations up to 2 m.

#### IMPACT VELOCITY

The iceberg impact velocity V is influenced by the prevailing current, wind, and waves. For simplicity, the impact velocity V in calm water (iceberg alone, no wind or waves) is taken here to be equal to the current velocity U:

$$V = U \quad (19)$$

This approximation is only needed with respect to the statistical descriptions of V and U, and is reasonably consistent with dynamic models of iceberg drift (e.g. [20] and [21]) when applied in the absence of waves and wind. Following data from Det

Norske Veritas [11], the current  $U$  is assumed to possess a lognormal distribution, with a mean  $\mu_U = 0.32$  m/sec and a standard deviation  $\sigma_U = 0.27$  m/sec.

## MAXIMUM FORCE

In calm water, the calculation of the maximum force  $F_M$  is implemented through an energy balance. The iceberg will be stopped when its kinetic energy is fully dissipated through ice crushing and structural damage deformation. Thus, this energy balance may be expressed as:

$$\frac{1}{2} M (1 + C_m) V^2 = \int_0^{x_c} F(x) dx + \int_0^{x_d} F(x) dx \quad (20)$$

where the iceberg mass  $M$  has been augmented by the added-mass coefficient  $C_m$  accounting for hydrodynamic effects. The first term in the right-hand side corresponds to the energy dissipated through ice crushing up to a penetration  $x_c$ , obtained from the force-penetration relationship. The second term corresponds to the energy dissipated through the local structural damage penetration  $x_d$ , and is taken into account only when the force exceeds a minimum force  $F_0$  required to produce damage.

The relationship between force and damage penetration has been estimated on the basis of a previous structural analyses of reinforced concrete elements at ultimate load. For the particular ice wall considered here, the results can be represented by a linear relationship up to a damage penetration of 1.5 m, according to

$$F(x) = F_0 + 1567 x_d R_{n5} \text{ (MN)} \quad (21)$$

with  $F_0 = 610$  MN. To account for the uncertainty in this estimate, the random variable  $R_{n5}$  is introduced, and is assumed to possess a lognormal distribution with a mean of 1 and a standard deviation  $V_f$ .

Given the geometry of the iceberg, its impact velocity, and the crushing pressure parameters, Eq. (20) can be solved iteratively to obtain the penetrations  $x_c$  and (if damage occurs)  $x_d$ . Once these are found, the maximum force  $F_M$  is obtained from the force-penetration relationship.

In practice an iceberg is likely to impact the structure in an eccentric manner. The influence of eccentric collisions has been considered, for example, by Bass, Gaskill and Riggs [22] and Salvalaggio and Rojansky [23]. In order to account for the possibility of eccentric collisions, the maximum force  $F_M$  calculated in the manner described is multiplied by an eccentricity reduction factor  $K_e \leq 1.0$ . Data from [23] are utilized to express  $K_e$  as

$$K_e = \cos^{0.228} \left( \frac{R_{n6} \pi}{2} \right) \quad (22)$$

in which  $R_{n6}$  is a random variable with a uniform distribution between 0 and 1. Thus,  $K_e$  varies from 1 for a head-on collision to 0 in the limit of the iceberg just making glancing contact with the structure.

The added-mass coefficient of an iceberg at impact,  $C_m$ , is determined by solving the boundary value problem corresponding to an iceberg undergoing small amplitude oscillations in otherwise still water. A description of the calculation procedure has been given by Isaacson and Cheung ([24], [25]). In general, the added-mass is frequency dependent, although it is customary to use a single value (usually the zero frequency value) when treating the iceberg impact problem. The added-mass of the iceberg depends on the submerged geometry, the water depth and the submerged geometry of any neighbouring structure (and thus it is a function of the iceberg distance from any such structure). The zero-frequency added-mass is estimated here for a range of iceberg parameters, both in open water and when in contact with the structure. However, for the range of iceberg sizes of interest, the added-mass is not strongly influenced by the proximity to the structure. A simple expression has been derived from numerical results obtained over a range of conditions:

$$C_m = 0.088 \left( \frac{D}{d} \right) \left( \frac{h}{d} \right) + 0.639 \left( \frac{h}{d} \right) - 0.100 \left( \frac{D}{d} \right) + 0.223 \quad (23)$$

## FORCE DUE TO ICEBERG COLLISION AND WAVES

Attention is now given to an iceberg collision in the presence of waves. The preceding descriptions of iceberg shape and size, crushing pressure, and added mass continue to apply. However, the maximum force on the structure is altered, partly because the impact velocity is changed, and partly because the wave force on the iceberg influences the iceberg force on the structure. Furthermore, the description of wave parameters must now reflect the sea state at the moment of collision, so that more commonly occurring wave conditions should be accounted for. These aspects are now considered further.

### WAVE PARAMETERS

For convenience, the description of more commonly occurring wave conditions is assumed to derive from measurements based on a specified recording interval, taken here as 6 hours. The corresponding probability distribution of the wave period  $T$  can be obtained from the distribution of the annual maximum period, given in Eq. (3), by raising the latter to the power of  $1/N$ , where  $N$  is the number of such recording intervals per year ( $N=1460$ ). The calculation details for the distribution of  $T$  corresponding to individual intervals is discussed by Foschi et al. [26].

### IMPACT VELOCITY

Although data for the open water drift velocity of icebergs is generally available, the actual velocity may differ from such data when storm waves are present and when the iceberg approaches the structure. Firstly, the waves give rise to a wave drift force which causes an increase in the iceberg velocity. This is likely not adequately accounted for in iceberg drift data, since such data generally pertains to commonly occurring wave conditions,

whereas design waves with a return period of order 100 years are of interest here. Secondly, since the wave drift force and iceberg added mass vary with distance from the structure, the iceberg velocity may be further modified through its equations of motion as it approaches the structure. Overall, it is expected that the impact velocity  $V$  depends on the current velocity  $U$ , the iceberg dimensions (characterized by its waterline diameter  $D$  and draft  $h$ ) and the prevailing wave conditions characterized by  $H_S$  or  $T$ .

A simple formulation for the iceberg impact velocity  $V$  may be developed by taking  $V$  to be proportional to the iceberg drift velocity in open water, denoted  $V_O$ , and adopting a suitable expression for the latter. Following Isaacson [8], an expression for  $V_O$  may be developed by equating the wave drift force to the current drag, taking the wave drift force coefficient to be proportional to  $D/L$  [8], where  $L$  the wave length, and using Eq. (4) to relate wave height and wave period. The above approach gives rise to the following expression for the impact velocity  $V$ :

$$V = U + \alpha gT \sqrt{\frac{D}{h}} \quad (24)$$

where  $\alpha$  is a constant and  $g$  is the acceleration of gravity. The value  $\alpha$  has been estimated by examining previous results and data for the open water velocity  $V_O$  (e.g. Lever and Sen, [5]) and using a numerical model to relate the impact velocity  $V$  to the open water velocity  $V_O$ . Thus, a wave diffraction-radiation analysis has been carried out for a series of conditions corresponding to the iceberg approaching the structure, using an extension to the computer program WELSAS. This provides solutions to the two-body diffraction-radiation problem of a floating iceberg approaching a fixed structure, as well as to the iceberg's equations of motion, and has been described by Isaacson [8]. For each such condition, the submerged surfaces of the structure and iceberg are discretized into a number of quadrilateral facets for a boundary element method analysis, in order to obtain the trajectory of the iceberg and the impact velocity  $V$ .

Overall, the foregoing procedure has indicated that Eq.(24) should be suitable for the conditions of interest here with the constant  $\alpha = 0.003$ .

#### MAXIMUM FORCE

In the presence of waves, the iceberg force on the structure,  $F(x)$ , is influenced in part by the wave force on the iceberg,  $F_w^{(i)}$ , Fig. 2. Thus, the calculation of  $F(x)$  is carried out by a direct integration of the equation of motion of the iceberg, rather than by a simple energy balance,

$$M(1 + C_m) \ddot{x} = -F(x) + F_w^{(i)} \quad (25)$$

with initial conditions  $x = 0$  and  $\dot{x} = V$  at  $t = 0$ , and an over-dot denoting a derivative with respect to time. The force  $F_w^{(i)}$  of the waves on the iceberg is expressed as

$$F_w^{(i)} = F_w^{*(i)} \cos(\omega t - \varepsilon) \quad (26)$$

where the angular frequency  $\omega = 2\pi/T$ , and the phase angle  $\varepsilon$  varies between 0 and  $2\pi$ .

The wave force amplitude  $F_w^{*(i)}$  has been calculated for a range of iceberg and wave parameters using an extension to the computer program WELSAS. The combined submerged surface of the iceberg in contact with the structure is discretized into a number of quadrilateral facets, and the magnitudes and phases of forces acting on the iceberg and structure are each computed by suitable integrations of the hydrodynamic pressure acting on the combined configuration. Since this approach in effect treats the iceberg and structure as a single contiguous body, rather than as two bodies in close proximity, no particular numerical difficulties are encountered. The numerical results are expressed as follows,

$$F_w^{*(i)} = 0.0172 h \frac{D^2}{T^2} H C_F^{(i)} \quad (27)$$

where  $F_w^{*(i)}$  is in MN, and  $H$  is the wave height in m. Details on

the force coefficient  $C_F^{(i)}$  are given in Foschi et al. [ ].

As already indicated in Fig. 2, the iceberg force on the structure,  $F(x)$ , is a nonlinear function of  $x$ . However, the iceberg will be stopped at values of  $x$  for which the function  $F(x)$  can be linearized as follows,

$$F(x) = K x \quad (28)$$

where  $K$  is the initial slope of the force-penetration relationship. With this linearization of the force  $F(x)$ , the equation of motion can be easily integrated in closed form. Of particular interest is the time  $t_0$  at which the velocity  $\dot{x}$  first vanishes (iceberg stopped), and the corresponding maximum penetration. The maximum iceberg force on the structure is obtained from the force-penetration relationship up to the point of maximum penetration.

Given the oscillatory character of the wave force on the iceberg,  $F_w^{(i)}$ , this force sometimes pushes the ice mass forwards and sometimes backwards. The phase angle  $\varepsilon$ , which controls the effect of this force at time  $t = 0$  (i.e. at the beginning of the collision), therefore has a substantial importance in the calculation of the exceedence probability. The FORM calculations are done conditional on specific values of the phase angle  $\varepsilon$ , with the total exceedence probability then calculated by integration over all phase angles from 0 to  $2\pi$ . The integration is facilitated by the simple probability density function of the uniform distribution for  $\varepsilon$ , using a Gaussian scheme with five integration points.

#### MODIFICATIONS TO PROBABILITY DISTRIBUTIONS FOR U AND L

The statistics for  $U$  and  $L$  correspond to all icebergs in open water. However, these statistics differ from those corresponding to impacting icebergs, since an iceberg's speed and size influence its probability of collision with the structure. Sanderson [18] has investigated this difference (see also Maes and Jordaan, [12])

and has shown that the speed and size probability distributions for colliding icebergs can be obtained as modifications of the corresponding distributions of all icebergs in open water. An application of Bayes' theorem enables the expressions of the modified probability density functions for U and L to be given as follows:

$$f^*(U) = f(U) \left( \frac{U}{\bar{U}} \right) \quad (29)$$

$$f^*(L) = f(L) \left( \frac{a+L}{a+\bar{L}} \right) \quad (30)$$

in which  $f(U)$  and  $f(L)$  are the corresponding open water probability density functions, and  $\bar{U}$  and  $\bar{L}$  are the corresponding mean values. It is seen that Eqs. (29) and (30) skew the original distributions so as to increase the chances of collision for bigger and faster icebergs.

## RESULTS

Table 1 shows a summary of the statistical parameters that have been used for the various specified random variables, except that the parameters for the iceberg draft  $h$  are determined by those of  $L$ . In order to verify the load combination factors specified in the Code, it is necessary to use these parameters to obtain the wave loads for an annual exceedence level of  $10^{-2}$  and both the iceberg alone and the iceberg plus waves loads at an annual exceedence level of  $10^{-4}$ .

Results for waves alone, at an exceedence probability of  $10^{-2}$  are shown in Table 2 for different storm durations. Forces for iceberg collision alone, at exceedence probabilities of  $10^{-2}$  and  $10^{-4}$ , are shown in Table 3. This Table indicates the influence of the annual exceedence probability, the iceberg arrival rate  $\nu$ , the modifications to the U and L distributions as described in Eqs. (29) and (30), and the ice crushing pressure threshold  $p_0$ . The Table indicates that the iceberg collision forces are strongly dependent on all four of these factors. In particular, it is apparent that the modifications to the distributions for U and L have a marked effect on the collision forces, and must be taken into account when the goal is to estimate the level of those forces for design.

Finally, Table 4 shows the results for an iceberg collision in the presence of waves. The annual exceedence probability is  $10^{-4}$ , as required by the Code. Once more, the Table indicates the influence of the iceberg arrival rate  $\nu$ , the modifications to the U and L distributions as described in Eqs. (29) and (30), and the ice crushing pressure threshold  $p_0$ . As in the case of an iceberg collision alone, all these factors have a marked influence on the maximum combined load. The results were obtained for a wave measurement recording interval  $\tau = 6$  hours, corresponding to  $N = 1,460$ . In fact, the use of different values of recording interval was examined, and found not to affect the results noticeably.

## LOAD COMBINATION FACTORS

The load combination factor  $\gamma$  is used in the CSA Code to determine a design value for the load due to a companion frequent environmental process (waves) acting in combination with a rare environmental event (iceberg collision). The load combination factor is defined in the Code in relation to the combined design load effect:

$$E = E_r + \gamma E_f \quad (31)$$

in which  $E$  is the combined load with an annual exceedence probability of  $10^{-4}$ ,  $E_r$  is the iceberg alone load with an annual exceedence probability of  $10^{-4}$ , and  $E_f$  is the wave load with an annual exceedence probability of  $10^{-2}$ . Thus the factor  $\gamma$  aims to achieve a combined wave-iceberg load with an annual risk of  $10^{-4}$ , just as it would be required in the case of an iceberg alone. The CSA code recommends:

$\gamma = 0.8$  for events stochastically dependent, and  
 $\gamma = 0.4$  for events stochastically independent.

and that iceberg impact in the presence of waves should be considered stochastically independent (i.e. the Code recommends  $\gamma = 0.4$ ).

Values of  $\gamma$  can be calculated from the results given in Tables 2, 3 and 4 and are shown in Table 5 for a storm duration of 8 hours. Differences for other durations are very small. Although the actual loads are substantially influenced by the iceberg arrival rate, the ice crushing pressure threshold  $p_0$  and the application of the distribution modifications according to Eqs. (29) and (30), the results in Table 5 suggest that the load combination factor itself is much more stable, and essentially only exhibits an effect of  $p_0$ . Since the iceberg collision force is interrelated with the effect of the waves, the combined event should be classified as dependent. However, the corresponding load combination factor of 0.80 is very much on the conservative side. Rather, the value of 0.40 can be conservatively applied for a range of conditions. Thus, this work confirms the recommendation in the current CSA code, except that the event should be classified as a dependent event.

## CONCLUSIONS

The analysis of loads due to an iceberg collision during a storm is presented. Mechanics models have been developed to provide forces on an offshore structure due to (i) waves alone, (ii) an iceberg alone, and (iii) the combination of waves with an iceberg. These have been combined with a reliability framework in order to obtain forces corresponding to specified risk levels.

The maximum combined load on the structure is associated with a number of factors. There is a significant increase of iceberg impact velocity due to the presence of waves. Likewise, the presence of the structure influences the wave force on the iceberg, which in turn influences the iceberg force on structure. One objective of this study has been to examine and recommend suitable load combination factors for combined iceberg-wave

loads on large offshore structures. At present, the CSA Code recommends that iceberg impact and waves should be considered stochastically independent, with a load combination factor  $\gamma = 0.4$ . The present study suggests that iceberg impact and waves should instead be considered stochastically dependent, but that the factor 0.40 can be conservatively used for a range of conditions.

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**Table 1 Random Variables Statistics**

Variable	Distribution	Characteristics
Iceberg length, L	Gamma	$\mu = 121.60$ m $\sigma = 56.70$ m
Iceberg draft, h	Beta	$\mu = 61.35$ m $\sigma = 12.38$ m Min. = 0.00 m Max. = 80.00 m
Current velocity, U	Lognormal	$\mu = 0.32$ m/sec $\sigma = 0.27$ m/sec
Wave period, T (annual maximum)	Extreme Type I	$\mu = 15.89$ sec $\sigma = 0.67$ sec
R <sub>n1</sub> , associated with model uncertainty	Normal	$\mu = 1.0$ $\sigma =$ (input)
R <sub>n4</sub> , associated with ice crushing pressure p	Normal	$\mu = 0.0$ $\sigma = 1.0$
R <sub>n5</sub> , associated with slope of load-damage deformation relationship	Lognormal	$\mu = 1.0$ $\sigma =$ (input)
R <sub>n6</sub> , associated with collision eccentricity	Uniform	Min. = 0.0 Max. = 1.0
R <sub>n7</sub> , associated with Rayleigh distribution for wave height H	Uniform	Min. = 0.0 Max. = 1.0

**Table 4 Wave-Iceberg Collision Forces**

Iceberg collision arrival rate (1/year)	U, L distribution adjustment	Maximum combined wave-iceberg load F <sub>M</sub> (MN)		
		(annual exceedence probability 10 <sup>-4</sup> )		
		p <sub>0</sub> = 2 MPa	p <sub>0</sub> = 4 MPa	p <sub>0</sub> = 6 MPa
0.04	No	2,305	3,120	3,797
0.08	No	2,592	3,604	4,394
0.20	No	3,088	4,331	5,287
1.00	No	4,175	5,880	7,189
0.08	Yes	4,220	5,952	7,281

**Table 2 Wave Forces**

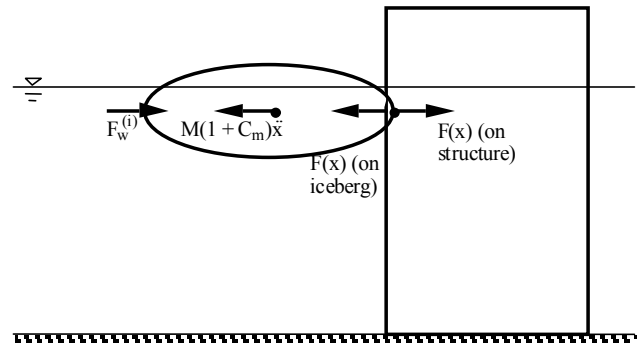
Annual exceedence probability	Maximum wave force F <sub>M</sub> (MN)		
	Storm Duration 4 hrs.	Storm Duration 8 hrs.	Storm Duration 16 hrs.
10 <sup>-2</sup>	2,625	2,745	2,825

**Table 3 Iceberg Collision Forces**

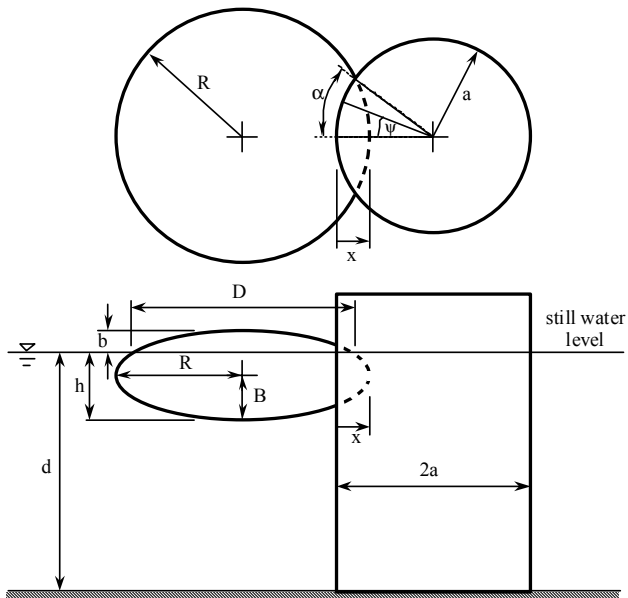
Iceberg collision arrival rate (1/year)	U, L distribution adjustment	Maximum iceberg force F <sub>M</sub> (MN)		
		(annual exceedence probability 10 <sup>-4</sup> )		
		p <sub>0</sub> = 2 MPa	p <sub>0</sub> = 4 MPa	p <sub>0</sub> = 6 MPa
0.04	No	1,605	2,276	2,792
0.08	No	1,932	2,740	3,360
0.20	No	2,425	3,442	4,224
1.00	No	3,484	4,952	6,080
0.08	Yes	3,588	5,106	6,270

**Table 5 Load Combination Factors**

Iceberg collision arrival rate (1/year)	U, L distribution adjustment	Load combination factor $\gamma$		
		$p_0 = 2$ MPa	$p_0 = 4$ MPa	$p_0 = 6$ MPa
0.04	No	0.26	0.31	0.37
0.08	No	0.24	0.31	0.38
0.20	No	0.24	0.32	0.39
1.00	No	0.25	0.34	0.40
0.08	Yes	0.23	0.31	0.37



**Figure 2 Wave-Iceberg Dynamic Equilibrium**



**Figure 1 Iceberg Geometry and Gravity Platform**